Study of the Laser Linewidth Influence on "lin || lin" Coherent Population Trapping

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We investigated, theoretically and experimentally, the influence of the laser spectrum (in particular the linewidth) on the Coherent Population Trapping (CPT) resonance in ^{87}Rb buffer gas cell. We investigated the case of linearly polarized laser fields. The experiments have been performed by using either a current-modulated Vertical Cavity Surface Emitting Laser (VCSEL) or a Phase-Locked-Laser (PLL) system. A model based on the density matrix approach has been used for the data interpretation. We found that, under experimental conditions suitable for compact clock applications, the fact that VCSELs have a broader spectrum than PLLs does not influence significantly the CPT signal amplitude. We demonstrate that a not negligible role is played by the destructive interference between different CPT excitation channels and propose further investigations to evaluate this contribution with respect to the other causes. Finally, we discuss the application of this scheme in compact atomic clocks.

I. Introduction

The applications of atomic clocks in telecommunications, navigation and positioning require the development of frequency standards for which not only the frequency stability but also the size and the power consumption are fundamental parameters. The CPT-based frequency standards are a promising alternative to the optical microwave double resonance scheme [1]. The absence of microwave resonator is an expected advantage of the CPT-based atomic clocks, and already allowed developing chip-scale devices [2]. A potential problem in CPT-based atomic clocks is the reduced contrast of the resonance signal, which affects directly the short-term frequency stability. During the last five years, several light-atom CPT-based interaction schemes have been proposed, to obtain higher resonance amplitude and a minimum linewidth [3]. In particular, in reference [4], the authors have shown that a significant increase of the CPT signal amplitude can be achieved by exciting transitions from the ground state with quantum numbers $F_g = 1, 2$ toward the excited state $F_e = 1$ by means of co-propagating laser waves with parallel linear polarization (the so-called "lin || lin" configuration). This condition can be satisfied in the D1 line of ^{87}Rb.

In this work, we study the influence of the laser spectrum (in particular the linewidth) in the lin || lin CPT resonance in a cell with pure ^{87}Rb isotope and 15 mbar of N_2 as buffer gas. We present the comparison between CPT resonances prepared by means of either a Phase-Locked-Laser (PLL) system or a current modulated Vertical Cavity Surface Emitting diode Laser (VCSEL). The current modulated VCSEL has a multi-frequency spectrum with broad linewidth (in our case 100 MHz), but is very compact and with low consumption. VCSELs are suitable for the applications in commercial devices. On the contrary, the PLL system has, in first approximation, a pure bichromatic field which spectral components have a linewidth of a a few tens of kHz only. However, it is bulkier than a VCSEL and has been used only in laboratory so far.

II. Excitation Scheme

The simplest system in which the CPT effect can be observed is a three-level atom formed by two ground states and one excited state, interacting with a two-frequency coherent radiation field. If the two frequencies match the two transitions of the three-level atom, the CPT effect occurs and the atom is trapped in a superposition of the two ground states that does not interact any more with the radiation field. This interaction scheme reminds the capital "lambda" letter of the Greek alphabet (Λ).

Let us consider the specific case of the D1 line of a ^{87}Rb atom, sketched in figure 1. When the atom is irradiated by a two-frequency laser field resonant with the two transitions ($F_g = 1 \rightarrow F_e = 1$) and ($F_g = 2 \rightarrow F_e = 1$), different Λ-schemes can be excited, depending on the polarization of the radiation field. When both frequency components of the laser field are *linearly* polarized in the *same* plane, and a static magnetic field parallel to the laser propagation vector is applied, the light-atom interaction scheme is the combination of σ^+ and σ^- transitions occurring simultaneously.

In the "conventional" CPT-based atomic clocks, the reference transition is between the sublevels $|F_g=1, m=0>$ and $|F_g=2, m=0>$ via (σ^+ - σ^+) or (σ^- - σ^-) excitation. In the lin || lin CPT, these two Λ-schemes occur simultaneously and may cancel each other: they do not contribute to the CPT signal. However, there are two additional Λ-schemes due to mixed (σ^+ - σ^-) excitations which correspond to the quadrupole ($|\Delta m| = 2$) transitions. They are represented in figure 1, and labeled "Λa" and "Λb". Λa involves the two pairs of ground state Zeeman sublevels $|F_g=1, m=-1> \leftrightarrow |F_g=2, m= 1>$, and Λb involves $|F_g=1, m=1> \leftrightarrow |F_g=2, m=-1>$. It is important to notice that the lin || lin CPT signal is suppressed at high buffer gas pressures, which is relevant for the potential of this approach in chip-scale atomic clocks (typically the pressure of buffer gas in a micro-cell is several hundreds of mbar).

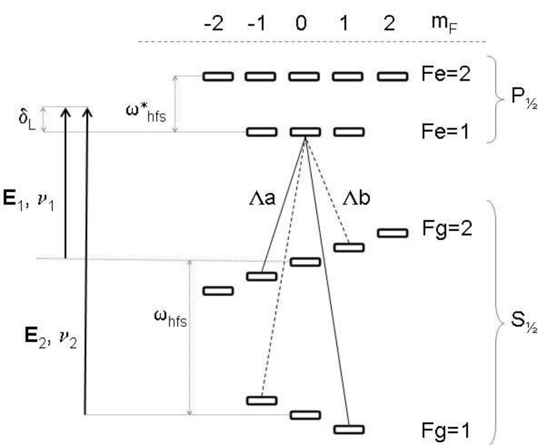

Figure 1: Scheme of the optically induced transitions in the ^{87}Rb atom, (F_e=1). To simplify the scheme, the "closed loop" involving the 2-photon clock transition is not shown. The signal that we propose to use as a clock reference is created by the two *Λ*-schemes quoted in the figure with the solid (Λa) and dashed lines (Λb).

The transition frequency shift of Λa and Λb with respect to the frequency of $|F_g=1, m=0> \leftrightarrow |F_g=2, m=0>$ "conventional" reference transition, is symmetric in the first order Zeeman effect (± 2.8 Hz/mG) and is due to the nuclear g-factor. As a consequence, for low values of longitudinal magnetic field intensity (few tens of mG), the frequency shift is small compared to the single resonance width, and the two dark resonances are unresolved. This degenerate CPT resonance can be used as frequency reference, and is the object of our study.

For data interpretation, we use a model based on the density matrix approach, discussed in detail in reference [5]. Therefore in this communication we describe only the crucial points of the model useful for the interpretation of the presented results. The model takes into account the hyperfine and Zeeman structure of the ^{87}Rb ground and excited states; as well as the pressure and the Doppler broadening of the hyperfine transitions. The laser is a multi-frequency field, and its one-photon detuning and linewidth are considered.

In the resonance condition, the atom can be treated as a quantum system with two large groups of energy levels: the ground and the excited states which are, in this context, the Zeeman sublevels of $S_{½}$ and $P_{½}$, respectively. Since the Doppler broadening is 540 MHz, which is comparable to ω^*_{hfs} = 817 MHz, our model assumes that the resonant field with frequency ν_i (i = 1, 2) interacts only with the i'th hyperfine sublevel of the ground state with $F_g = i$ but with both hyperfine sublevels of the excited state, F_e=1, 2. The model allows to compute the population of the ground (ρ_{gg}) and of the excited (ρ_{exc}) state by means of a set of differential equations. The complete form of the equations have been presented and discussed in reference [5]. It is important to notice that the total population of the excited states, ρ_{exc}, allows to calculate the photo-detector current (j), which is experimentally recordable, by means of equation:

$$j = (P_0 - \delta P) \cdot \frac{e}{\hbar\omega} K = \left(\frac{P_0}{\hbar\omega} - \gamma \cdot N \cdot \rho_{exc} \right) Ke \quad (1)$$

In Eq. (1), P_0 is the laser power incident on the cell; $\delta P = (\gamma \hbar\omega N)$ is the power absorbed in the cell, $K \approx 1$ is the quantum efficiency of the photo-detector, ω is the optical transitions frequency; γ is the decay rate for the excited state; and N is the number of the atoms irradiated by the laser beam.

ρ_{exc} depends on the optical relaxation, γ', that can be written as:

$$\gamma' = \frac{\gamma_{sp} + \gamma_{coll} + \Gamma_{laser}}{2} \quad (2)$$

In eq. (2) Γ_{laser} is the laser linewidth, γ_{sp} is the spontaneous relaxation rate and γ_{coll} is the pressure broadening of the optical transition.

III. EXPERIMENTAL SETUP AND CHARACTERISATION OF THE LASER SOURCES

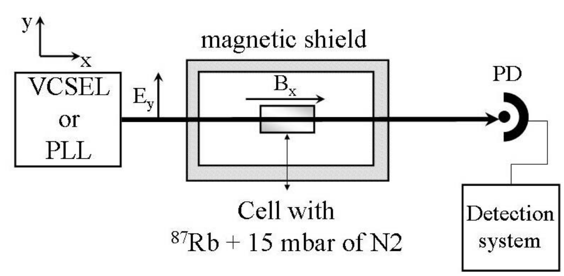

Figure 2: Block diagram of the experimental setup.

The block diagram of our experimental setup is shown in figure 2: the core is a glass cell (length 2.4 cm and diameter 1.5 cm) containing pure ^{87}Rb isotope and 15 mbar of N_2 as buffer gas. The cell temperature was stabilized at 68°C. The values of the pressure and Doppler broadening of the Dl line are given in table 1.

In order to avoid the influence of spurious magnetic fields, the cell was inserted in a magnetic shield and a solenoid provides a static magnetic field parallel to the laser beam propagation vector (B_{long}). B_{long} was typically 30 mG and the frequency shift between the Λa and Λb CPT signals was therefore of about 170 Hz, which is much less than the CPT linewidth (typically in the kHz range). The laser light passed through the cell and was collected on a photo-detector. In our experiments, the CPT amplitude was large enough to allow the direct monitoring of the dc signal. However, to extend the measurement range, the absorption and dispersion spectra from a dual-phase lock-in amplifier have been recorded simultaneously. It is worth to notice that the dispersive lock-in signal can be used as frequency reference in atomic clocks.

Table I: Broadening of the Rb D1 line for the experimental conditions

Buffer gas pressure, mbar	***15***
Pressure broadening γ_{coll}, MHz	210
Doppler broadening at 68 C, MHz	515

Both laser sources have the same beam profile (Gaussian) and a beam diameter of 2 mm. The maximum power emitted by the VCSEL is 0.3 mW, while the PLL has a larger power range (up to a few mW). In the following the important differences in the spectrum between VCSEL and PLL are discussed.

Current modulated VCSEL

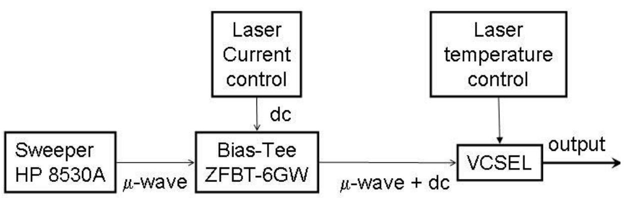

Figure 3: Block diagram of the VCSEL source. The laser is modulated with a commercial function generator and the modulation is directly injected on the laser chip current.

In figure 3, the block diagram of the modulated VCSEL (single mode VCSEL chip emitting around 795 nm) used during the experiment is shown: the laser is controlled in temperature and current by low-noise homemade controls. The choice of using a VCSEL is due to the large current tuning coefficient of this type of laser [6], for our VCSEL we measured 143 GHz/mA. This intrinsic characteristic, due to the short cavity of the VCSEL, allows reaching a sufficient modulation index at high frequencies even at a relatively small microwave power (few mW). The microwave modulation is injected in the laser chip by means of an external connection (Bias-Tee). The VCSEL used in this experiment has a linewidth of about 100 MHz, measured by means of a beat signal with an ECDL (Extended Cavity Diode Laser), as shown on figure 4. The VCSEL is modulated at the frequency of 3.417 GHz (i.e. half of ground state hyperfine separation in ^{87}Rb) with a power of 10 dBm. The dark state is obtained when two frequencies of the multi-frequency spectrum match the ground state hyperfine splitting. In general, the injection current modulation of a diode laser results in both amplitude and frequency modulation. We did not measure directly the spectrum of the modulated VCSEL, but we estimated it using the ^{87}Rb one-photon absorption for different modulation frequencies and amplitudes. In our spectrum we can not observe any effect related to the amplitude modulation, so in first approximation we neglect this effect and the frequency modulation index is about 1.8. In this condition, the 98% of the total laser power is distributed in carrier, 1st and 2nd order side-bands, according to the Bessel J functions. In our case the 1st order side-bands (about 68% of the total power) are tuned resonant with the atomic transitions contributing to the CPT signal. The other frequencies essentially do not contribute to the CPT signal but increase the dc "background" level on the photo-detector and could contribute to the one-photon absorption process.

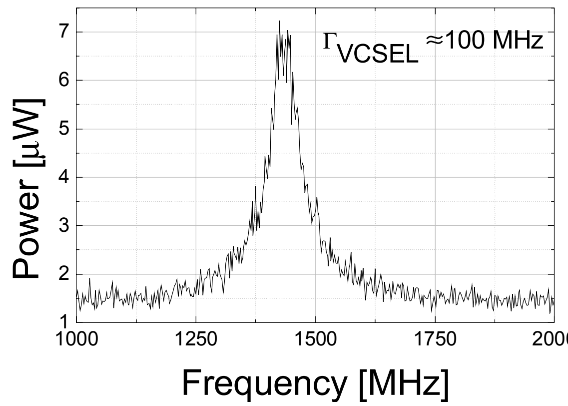

Figure 4: Beat signal between VCSEL and an ECDL. These measurements allow estimating the laser linewidth (Γ_{VCSEL}~100 MHz).

Phase-Locked-Laser (PLL) system

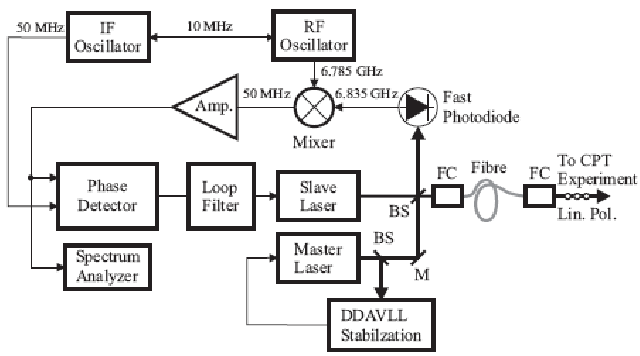

Figure 5: Block diagram of the PLL system.

The experimental setup used for CPT-dark resonance preparation with the optical phase lock loop (OPLL)-system is shown in figure 5. Here the master laser is stabilized on the ^{87}Rb D_1 $5^2S_{1/2}$ F = 2 -> $5^2P_{1/2}$ F' = 1 transition using a DAVLL-scheme [7]. A linewidth of 40 kHz root mean square (rms) is achieved in combination with high speed servo electronics. The light beams from the master and the slave extended cavity laser are superposed on a fast photodiode (New Focus Mod. 1537) which detects the 6.8 GHz beat signal required for CPT-experiments in ^{87}Rb. After amplification, the beat signal is compared to a reference frequency signal from the Intermediate Frequency (IF-) oscillator at the desired difference frequency in a two step process (figure 5):

(i) In a 7 GHz double balanced ring mixer the beat signal from the fast photodiode is down converted into the 50 MHz IF-band by a radio frequency (RF-) oscillator.
(ii) A digital phase/frequency detector provides an output signal proportional to the phase difference between the down converted beat-note and the IF-oscillator.

The IF-oscillator is by itself coupled to the high stability 10 MHz frequency reference of the RF-oscillator. In this way the high stability and phase coherence is also guaranteed for the beat-signal phase.

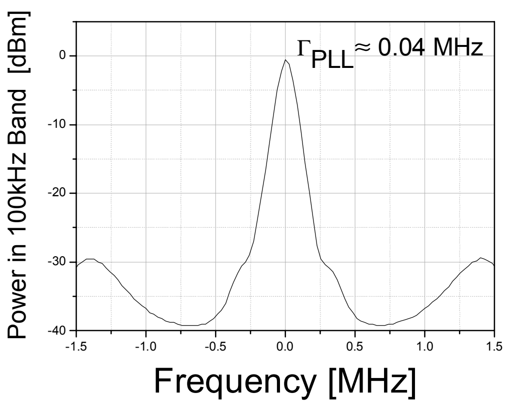

Figure 6: Beat-spectrum of the phase coherent light field to estimate the laser linewidth (Γ_{PLL}~0.04 MHz).

In CPT experiments, phase excursions of more than $\pi/2$ during the life time of the ground state coherence, typically milliseconds, lead to a destruction of the coherence and a broadening of the resonance line [8]. On the other hand, long term stability is required for spectra with long integration time and wide span. Therefore the advantages of an analog (low phase noise) and a digital phase detector (high stability and great capture range) are combined in this setup. In this way the low phase noise, under (phase) locked conditions, enables experiments on narrow CPT-Resonances with a Signal to Noise Ratio (SNR) typically better than 100.

The performance of the loop can be derived from the IF power-spectrum (see Spectrum Analyzer in figure 5) of the beat note [9] in figure 6. By means of this a total rms phase error of 0.04 rad (rms) is achieved.

In the case of PLL, the frequency components of the bichromatic electromagnetic field used in the CPT-experiments are separated by 6.835 GHz. The intensities as well as the linear polarization state are selected to be the same for both frequencies. Finally, to avoid a residual broadening of the CPT-resonances due to a wave vector mismatch [10] a polarization maintaining single mode fiber is used to reach perfect collinear wave vectors $\mathbf{k_1}$ and $\mathbf{k_2}$ of both frequency components.

The most significant differences on the spectrum of the PLL system and the current modulated VCSEL are summarized in table II and have been schematically represented in figure 7.

Table II: comparison between the spectral characteristics of PLL system and current modulated VCSEL.

Laser system	*PLL*	*VCSEL*
Number of sidebands in the spectrum	2	~5
Distance between 2 consecutive frequency, GHz	~6.8	~3.4
Linewidth Γ_{laser} , MHz	0.04	100

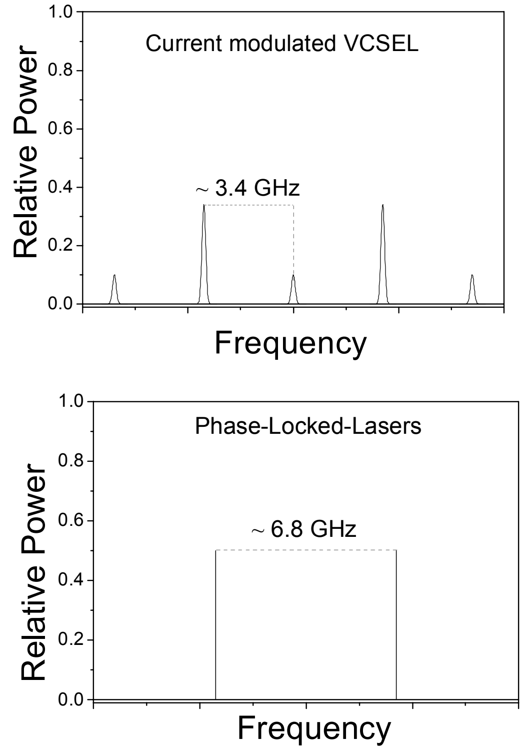

Figure 7: Schematic view of the relative power distribution for the two light source used in the experiments. In the upper figure is sketched the multi-frequency spectrum of the current modulated VCSEL: 68% of the total power is in the resonant side-bands. In the lower figure is sketched the case of the bichromatic PLL spectrum, where the total power is equally distributed in the two resonant frequencies.

IV. CPT SIGNAL AMPLITUDE VERSUS LASER LINEWIDTH : EXPERIMEANTAL RESULTS AND DISCUSSION

The expected short term instability of an atomic frequency standard is inversely proportional to the Discriminator slope (D) of the reference signal; and D is proportional to the ratio between the amplitude and the linewidth of the atomic resonance signal. In this communication we analyze the dependence of the lin || lin CPT amplitude and linewidth on the laser spectrum by comparing the measurements done with the two laser systems described above.

We define the relative CPT amplitude as:

$$A := \frac{j_{CPT} - j_{nr}}{j_{nr}} \tag{4}$$

In eq. (4) j_{CPT} is the photo-detector currents at the maximum of the CPT resonance and j_{nr} is the photo-detector current due to the resonant light out of CPT resonance but near the working point, i.e. in the Rb optical resonance. The relative CPT amplitude, A, corresponds to the contrast for the PLL experiment while for the experiments with a VCSEL the contribution of off-resonant frequency is subtracted. In other words the CPT contrast in the VCSEL is reduced by the off-resonant light.

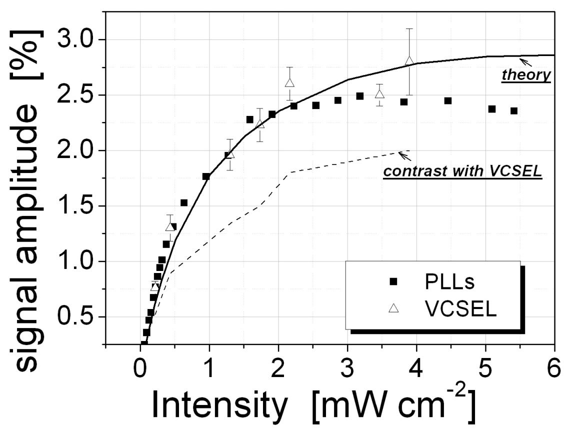

Figure 8: Relative CPT amplitude versus the laser intensity: the difference in the relative amplitude recorded with PLL (square points) and VCSEL (triangular points) is negligible. The experimental results is well reproducible by the model (solid line). The relative amplitude in the case of PLL experiment correspond to the CPT contrast for this reason the CPT contrast obtained with the VCSEL is also reported in the plot (dashed line).

We can observe in figure 8 that the difference in the CPT amplitude obtained with the VCSEL (triangular points) and the PLL (square points) is negligible and well reproducible by the model (solid line), but the contrast in the case of VCSEL experiment (dotted line in figure 8) is smaller than the contrast obtained with PLL (triangular points) due to the contribution of off-resonant light.

In our experiments the laser linewidth, for both PLL and VCSEL, is smaller than the homogenous broadening of the D1 line ($\Gamma_{PLL} \ll \Gamma_{VCSEL} < \gamma_{coll}$), see Table I and II. In [6] the authors recorded, for the D2 line of Cs, a smaller CPT amplitude when the laser linewidth is larger than the homogeneous linewidth because the CPT resonance is excited in atoms belonging to several velocity classes. The inverse consideration can be applied in our experiments, but a difference in the CPT signal amplitude is expected in any case, essentially due to the multi-frequency spectrum of the VCSEL. In fact the frequencies out of resonance contribute to the one-photon absorption in the Doppler broadened transition inducing losses in the CPT signal, especially the carrier frequency which is tuned between the two hyperfine ground states. To explain our results (figure 8) the influence of transitions towards F_e=2 have to be taken into account. These transitions seems to be not negligible even if the laser is tuned resonant to F_e=1. The influence of the excited hyperfine states in the σ–σ CPT signal has been showed for Cs atoms in [11].

Let us analyze into more details our case. For simplicity, we first discuss the case of Λa (see figure 1). This example can be extended to Λb and to the combines Λa and Λb.

Figure 9 is obtained by adding to figure 1 the σ^+-σ^- transitions towards F_e=2 that should be taken into account for the sake of completeness in the description of the interaction.

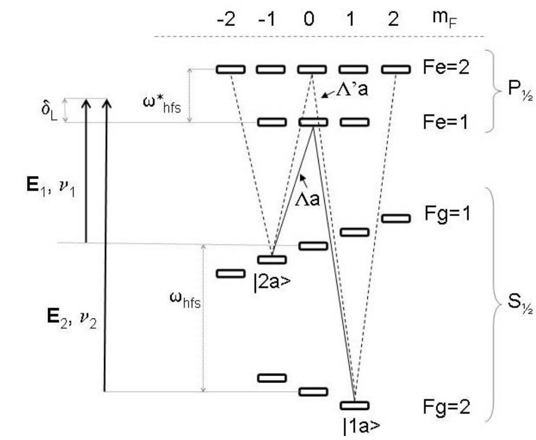

Figure 9: Scheme of the optically induced transitions in the ^{87}Rb atom, considering both hyperfine extited level F_e=1 and F_e=2. We focus our attention on the Λa scheme (already shown in figure 1): in the plot are represented the transition towards F_e=2, that could influence the recorded signal even if the laser is tuned resonant with the group of transition starting from F_e=1.

Apart from the losses due to the transitions towards $|F_e=2, m_F=\pm 2\rangle$, one can notice that there is a closed excitation contour formed by the scheme Λa (toward F_e=1) and Λ'a (toward F_e=2). In similar systems the existence of CPT is directly connected to the phase relations between the Rabi frequencies along the excitation contour. The form of "dark states" $|\Psi_{\Lambda'a}\rangle$ and $|\Psi_{\Lambda a}\rangle$ has been calculated in [5] and these states are orthogonal ($\langle\Psi_{\Lambda a}|\Psi_{\Lambda'a}\rangle = 0$). Therefore, if they are both excited, the CPT signal amplitude is reduced due to their destructive interference. In principle, if their contributions are equal, the CPT signal cancels.

To quantify the influence of F_e=2, a factor (κ) can be introduced and estimated by using the model. Quantitatively, κ depends on the ratio ω^*_{hfs}-to-γ_{coll} and on the laser detuning (δ_L). According to the model, the influence of the transitions towards F_e=2 seems to be negligible if $\kappa < 1/70$, while in our experiments $\kappa \sim 1/20$.

In synthesis, it appears that under experimental conditions suitable for clock application, the spectrum of the VCSEL does not affect significantly the CPT amplitude. On the other hand, no difference in the CPT linewidth (typically a few kHz) is expected and recorded because the broadening is mainly due to the optical pumping rate.

V. Application in compact frequency standard: noise and expected short term stability

In the first part of this communication, we have shown that, under suitable conditions, a broad light source does not affect the CPT parameters (amplitude and linewidth); but the short term stability of an atomic clock is also related to the detection noise.

In the case of PLL, the coupling into the single mode fiber (figure 5) is critical and can lead to an excess of noise ranging from 1 to 10 times the shot noise (N_s) level of the photo-detector. However, it is possible to align the fiber so that the N_s level is reached.

In the case of the VCSEL setup, our analysis, allowed to minimize the detection noise in view of the applications in high performance compact atomic clocks. First, we optimized the microwave modulation conditions and we verified that the current modulation did not add any extra-noise. The results of our analysis are presented in figure 10. The detection noise has been measured varying the one-photon detuning of the VCSEL. We observe that the noise level depends on the laser frequency detuning: when the laser is resonant with the Rb absorption the noise level increases significantly.

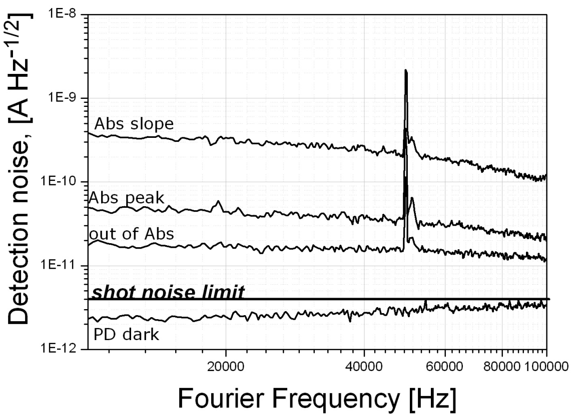

Figure 10: Detection noise measured for the dark photo-detector, for the laser field tuned out of the ^{87}Rb D1 line absorption, and for the laser tuned to the maximum and to the slope of ^{87}Rb D1 line absorption. The dominant contribution seems to be due to the laser FM-to-AM conversion.

After our preliminarily evaluation the dominant effect seems to be the laser FM-to-AM noise conversion and a relevant contribution to the detection noise is due to the laser side-bands which do not contribute to the clock signal.

In conditions optimized for clock application the expected short term stability for lin || lin CPT prepared using current modulated VCSEL, is about $6 \cdot 10^{-12}\ \tau^{-½}$, estimated by the expression for the Allan deviation presented in ref [12]. One should consider that in VCSEL experiment the detection noise is dominated by the laser noise and the estimated N_s, is one order of magnitude smaller than the measured noise. On the contrary, the PLL system can be N_s limited and the expected N_s value should be even less than the VCSEL one (due to the absence of off-resonant light). Finally, one can expect to reach better short term stability (a few 10^{-13} at 1 sec) by using PLL system, N_s limited and in the lin || lin CPT configuration with a lower buffer gas pressure.

VI. Conclusion

In this communication we present our results on the comparison between lin||lin CPT prepared by using either a current modulated VCSEL or a PLL system. Even if the spectrum (and in particular the linewidth) of the two laser systems is considerably different the CPT amplitude and linewidth obtained in cell containing pure ^{87}Rb isotope and 15 mbar of N_2 as buffer gas are not significantly different. For fully understanding this effect, the quantum interference between multi-CPT channels simultaneously activated because of the homogenous broadening ($\gamma_{coll} \approx$ 210 MHz) should be taken into account. In order to validate our results and quantify the influence of the different effect, further experiments are in progress to verify the behavior when the opposite condition is satisfied: $\Gamma_{PLL} < \gamma_{coll} \leq \Gamma_{VCSEL}$. Finally we discussed the potential of VCSELs in compact frequency standard: the CPT discriminator signal is not limited by using a broad laser source but the expected performances are essentially limited by the detection noise.

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